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What kinds of mistakes do students make after learning exponential functions online?: A case study

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ABSTRACT: Due to students' mistakes in comprehending exponential functions, the lifting problem results in low learning outcomes. This study attempts to confirm the accuracy of this and determine the types and reasons for student errors. An approach known as descriptive qualitative is used in this case study research. Data was gathered from the outcomes of tests using the exponential function. 12 of the class's 32 pupils were chosen as the research subjects based on observations. It was discovered, nonetheless, that some pupils also committed errors in the Transformation, Process Skills, and Coding categories. In general, it may be said that students make mistakes because they don't understand the idea of exponential functions and their graphs, as well as because they make blunders while using fractional exponents. Bold education is one of the causes of this problem. Therefore, teachers must really check the results of students' work so that it is clear where the mistakes made by their students are clearly visible so that they can improve the mathematics learning process in the future.

KEYWORDS: Case study; exponent; newman's error analysis

Introduction

Math proficiency is a requirement for learning in school. They may overcome issues that arise during studying and in their daily work by knowing mathematics (Boyd & Ash, 2018; Widakdo, 2017). Students whoa are proficient in mathematics will be able to reason clearly and systematically (Cai et al., 2015). This way of thinking develops as a result of how frequently students arrive at the right solution to a mathematical issue (Korkmaz et al., 2017; Rittle-Johnson et al., 2019). The more issues you work on, the more practice pupils will get in math. While attempting to answer these questions, it is typical for students to make mistakes (Li et al., 2017; Weintrop & Wilensky, 2015). However, further investigation is required to determine the cause if the majority of pupils make mistakes when attempting to solve mathematical problems Calculation errors are caused by misconceptions (Abdullah et al., 2015) and not being thorough (Willingham, 2021).

Continuous errors experienced indicate that students are experiencing learning difficulties (Wijaya et al., 2015). The help that students experience when learning mathematics can be traced through the mistakes they make when solving mathematics problems (Herscovics, 2018). the assistance experienced by students when learning mathematics takes the form of obstacles, distractions and obstacles when absorbing various information (Chinn, 2020). Continuous completion will have an impact on poor mathematics learning outcomes (Chen et al., 2016; Van Der Kleij et al., 2015). These bad results will occur continuously if they are not corrected and given attention. Errors are seen as a form of deviation from predetermined conditions (Bayar et al., 2015). One of the mathematics materials where students

experience a lot of errors when solving problems is experimental.

The fundamentals of exponentials are crucial in daily life since numerous issues, including population growth, the rise of bank money, and other issues, can be computed and approximated exponentially (OECD, 2018). The COVID-19 pandemic's exponential growth and the risks of underestimating it are the most prominent examples of this today (Schonger & Sele, 2021). Students can be taught exponential functions in a variety of ways, such as by employing engaging learning strategies (Naik, 2017), methods that differentiate instruction based on the needs of the students (Boss & Larmer, 2018), or engaging learning modules (Sahin et al., 2015). The definition of an exponential function, its mathematical definition, and its characteristics are among the topics covered in the learning of exponential functions (Lagos, 2020). In the module he developed, Lagos also explained the activities carried out by students in the classroom, namely in groups and individually. The group activity is to form an exponential function using the concept of binary fission in bacteria and combine it at the end of learning, while the individual task is to graph the exponential function with dengan b>0 and b<0 (Lagos, 2020).

The concept of the exponent $a^n = a \times a \times a \times \ldots \times a$ where a is a factor of n, is related to the concept of multiplication in elementary school, then the expansion and deepening of the material in junior high school, and is related to the concept of sets, relations, ordered pairs, and function (Haji, 2019; Vergnaud, 2016). A simple problem that students must understand is what exponentiality is, misunderstandings often arise. help and errors in solving mathematical problems arise when mathematical concepts are not mastered by students (Abdullah et al., 2015). The two main reasons why students can make procedural, factual or conceptual errors are (1) students' lack of knowledge, and (2) students' poor attention and carelessness (Hansen et al., 2020). Exponential function material is meeting material to study logarithmic functions, the two are interrelated. As we know, the logarithmic function is the inverse (reverse) of the exponential function (Haans et al., 2016). Almost every process of solving a logarithm problem requires rewriting the problem in exponential form (Triana & Zubainur, 2019). Exponentials are also a basic basis for understanding and deepening other algebra material. Therefore, students must master exponentiation so that there are no problems when studying algebra material that requires the concept of exponents.

In one study, students were asked to change and change an equation form $(3x^2)$ or $(3 + x)^2$, but students experienced problems because they were unable to distinguish the different stages of completion from the teacher's explanation (Adler & Alshwaikh, 2019). Students who experience misconceptions in solving numbers are based on aspects of generalization, notation, specialization, and language (Nurkamilah & Afriansyah, 2021). Students often make mistakes in drawing or sketching various functions in Cartesian coordinates (Takači et al., 2015), especially exponential functions (Triana & Zubainur, 2019). Errors depict graphs of functions associated with two different quadratic approximations of exponential functions (Gordon & Yang, 2016). This study discusses the simple problem of the general form of the exponential function $(f(x) = ka^x)$, slightly different from the exponential function $(f(x) = e^x)$ Mistakes and carelessness made by students can be explained using Newman's Error Analysis procedure.

Newman's Error Analysis (NEA) can be used as an appropriate assessment strategy to encourage focus on word problems and diagnostic assessment by teachers (White, 2018). Newman's research results stated

that many students had difficulty with semantic structures, vocabulary, and mathematical symbols compared to ordinary mathematical algorithms (White, 2018). NEA also provides Directions where teachers can target effective teaching strategies to overcome them (Khalo & Bayaga, 2015). Various kinds of problem analysis research using NEA procedures, including error analysis by reviewing students' problem-solving abilities (Rohmah & Sutiarso, 2018), students' errors in solving space and shape problems in PISA problems (Sumule et al., 2018), errors in algebra problem solving (Son et al., 2020), student errors in solving arithmetic problems among introverted students (Trapsilasiwi et al., 2021), student errors in solving triangular and quadrilateral plane problems (Asriyani et al., 2020), errors in ratio and proportion based on gender differences (Hanifah et al., 2020), listening and developing thinking skills (Alhassora et al., 2017), comparing accuracy between students who memorize and do not memorize the Qur'an in solving linear programming problems (Mukminah & Riana, 2020). Analysis of researched errors to find solutions when students experience misconceptions in various parts when working on mathematics problems.

Error kinds and causes can be distinguished procedurally, which means that each error has a unique cause. The error analysis method employed is called Newman's Error Analysis (NEA), and it uses reading, comprehension, transformation, processing skills, and encoding faults as error indicators (Newman, 1977); (1) Reading mistakes cause pupils to misread words or symbols, which prevents them from understanding difficulties (Kristianto & Saputro, 2019; Zamzam & Patricia, 2018). If students are unable to read, understand, and grasp the meaning of each word, phrase, or problem sign, they are urged to make mistakes in their reading (Agustiani, 2021; Oktafia et al., 2020). (2) Misunderstanding and errors in understanding occur as a result of pupils failing to comprehend and interpret important terms in the subject matter and failing to read the questions carefully (Sumule et al. al., 2018). Apart from that, one of the influencing factors is that students understand the context of the question but are unable to write the meaning correctly (Riastuti et al., 2017). (3) Transformation errors, transformation errors occur because students cannot determine the formulation or plan a solution to solve the problem (Sumule et al., 2018).

Forgetting material and formulas and errors in determining the mathematical operations used can also be other factors that can influence transformation process errors. Failing to identify mathematical operations and create or convert them into mathematical models (Asriyani et al., 2020). (4) Process skill errors, students do not know the completion steps, experience errors in calculations (Asriyani et al., 2020). Students are less careful and cannot perform arithmetic operations well (Winarso et al., 2022). (5) Enconding error, even though they have completed the mathematics assignment correctly and correctly, the student fails to provide the written answer obtained (Wardhani & Argaswari, 2022). Usually students do not check the results of their work before collecting them (Priliawati et al., 2019) and cannot show answers and final conclusions and do not write them completely (Asriyani et al., 2020). Completion of the last three steps (Transformation, Process Skill and Encoding) indicates that students have successfully carried out the mathematical processes required to complete the assignment (Sovia & Herman, 2019).

This research was conducted during the COVID-19 pandemic. During this pandemic, the learning perspective was changed to dare learning (online). Meanwhile, the research subject's location and school have been holding learning per shift, meaning that half of the students in the class are scheduled to go to class to study alternately. Meanwhile, other students who do not enter class can listen to the lesson using Google Meet media. Learning using Google Meet will be combined with the online Geogebra application. The aim of the research is to determine the types of errors and causes of students' errors in solving exponential problems based on Newman's Error Analysis procedure. Analyzing these errors can minimize the occurrence of similar errors in subsequent material. Knowing the mistakes students experience can help teachers to master their students. Teachers must know their student's abilities and intelligence levels in order to be able to create problem instructions or design learning tools that are useful in minimizing students' errors in understanding mathematical concepts. Apart from that, the teacher can convey or confirm back to students the results of the work/answers they have written. This aims to ensure that the same mistakes are not repeated in the future.

Research methods

This study employs a descriptive qualitative methodology and is a case study. a description of how students think, the kinds of mistakes they make, and the reasons behind those mistakes (Johnson & Christensen, 2014). Triangulation is used in data analysis to verify the correctness of results and data interpretation. It is employed as a method of bolstering evidence derived from many types of data. In order to ensure that the results and interpretation of data from the same source, namely student work results and interviews, are accurate, this study used data analysis.

The first step is to observe one of Langsa City's high schools in order to understand how the exponential learning process functions there. The class X math instructor was the subject of a brief interview. Next, the determination of the subject was determined from the results of the interview with the teacher. The research subjects were 12 students selected from 32 students in class Data was collected by written tests and interviews. Given 10 questions about exponents in short answer form.

Giving tests and questionnaires to students will be explained using two procedures, namely Newman's error analysis. Students who have completed their work are asked/interviewed several questions regarding the reasons for the answers written. The next step in analyzing errors with the Newman procedure requires the researcher to ask the subject to provide responses to questions and perform several tasks.

Research findings

Results of Newman's Error Analysis error analysis of 12 class X students in solving exponential problems. The percentage of students who make mistakes solving exponential problems based on Newman's Error Analysis indicators is as follows.

In Table 1, it can be seen that the most common mistakes students made were comprehension errors as much as 67%. Many people also make mistakes in solving question number 2. Below we will explain some of the students' mistakes that occurred, namely:

1. In writing exponent form, a student (S4) wrote the rank $5^{\frac{1}{2}}$ with $5\frac{1}{2}$, this was a very

fatal mistake. Because there are two different concepts between the two, of course, the results obtained are not correct. This type of error includes reading errors. Students fail to recognize words or symbols, and students do not find the correct exponent form. This is because students incorrectly identify exponent numbers.

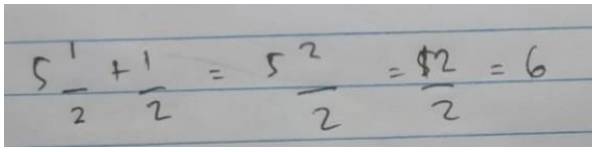

Figure 1. Errors in writing exponential forms

2. Adding powers, students incorrectly added powers in fraction form, it was found that several students (S1, S2, S4, S8, S9, and S11) made mistakes in the same part. When multiplying two numbers with the same exponent, the powers of each number must be added.

The stage of working on the concept of multiplying exponentials is correct, a^m×a^n=a^(m+n), however 50% of the subjects are wrong in adding the ranks (1/2+1/2=2/4). Learning fractions begins in elementary school and continues with this material in junior high school. One of them stated, "Ma'am, we at junior high school also had time to learn online, we had difficulty understanding what the teacher explained using the brave learning system." It can be seen from this problem that the type of error made by students is a comprehension error because students understand the context of the question but cannot memorize the meaning correctly.

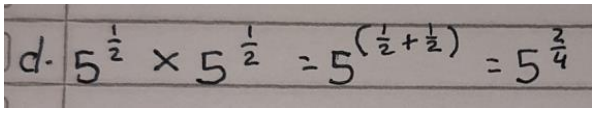

Figure 2. Multiplication of Exponential Numbers

3. Division of fractions, from the results of S10 answers, students were careless in operating the division of a number of fractions. The error begins with changing 3^{-4} and 3^2 into fraction form. This time the students incorrectly operated partial division. The mistake made by the S10 was very simple but had a big impact. The reason is that students forget the material and still seem confused about the concept of fractions that they have studied in junior high school. This case is a type of transformation error.

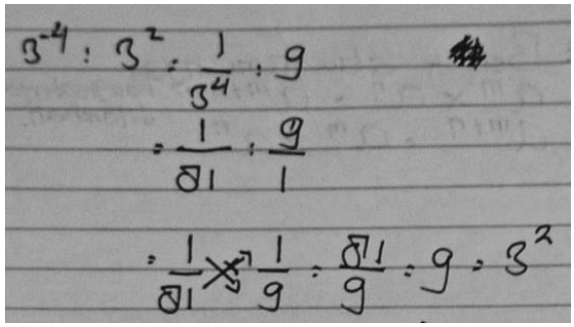

Figure 3. Division of Numbers in Fraction Form

4. Division properties of exponential functions, S10 also misunderstands the properties of exponentials, $a^m : a^n = a^{m-n}$. In the problem below, the number 6 which should be the numerator for the exponent changes its meaning to become the numerator for the abscissa (base number).

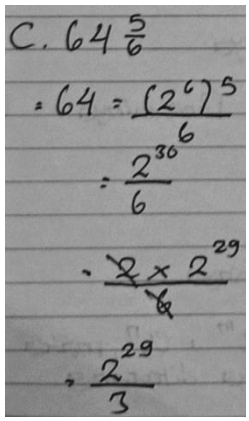

Figure 4. Mistakes in interpreting rank

Table 1. Percentage of student answers base on newman's error analysis

Question Number	Indicator Newman's Error Analysis				
	Reading	Comprehension	Transformation	Process Skill	Enconding
1	0	0,5	0,08	0	0
2	0,25	0,08	0,08	0,17	0,08
3	0,17	0,08	0	0	0
Total	0,42	0,67	0,17	0,17	0,08

When asked why he answered that way, SI0 explained "Change 64 into a power number then multiply the power by 5 and then take 1 out of the power and divide the result by 6, giving a result like this." Based on this explanation, it can be seen that students have forgotten or do not know the concept of division into ranks. Process skill errors occurred in this case, where students incorrectly applied the procedure for dividing multilevel numbers. The reason is that students are not careful and do not provide answers according to the context of the question. Occurs in the same error type with different cases. Students did not write down the work process, S1 answered with the same results but could not provide reasons for his explanation. The results obtained do not undergo any processing. Students also do not provide reasons as responsibility for their completion.

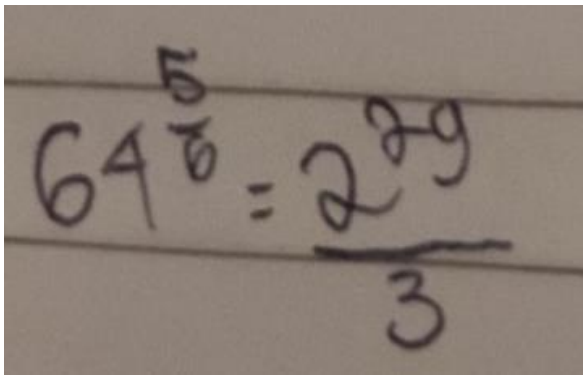

Figure 5. Not writing down the settlement process

5. Write down the results of multilevel numbers in decimal form, there are also students who do not use the concept of exponents in solving them, such as S4 solving problems in exponent form with integers and decimals. It can be seen that the student is less critical in analyzing questions, he does not apply the concept of exponents. Two types of errors are contained in the following problem.

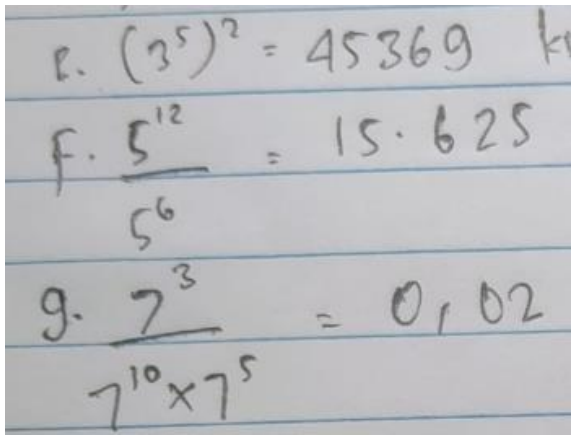

Figure 6. Writing the results of exponent numbers using whole numbers and decimals

The picture above shows students making two types of errors, comprehension errors and transformation errors. Where students do not properly grasp the problem information contained in the questions, do not understand the material (Sumule et al., 2018), and cannot complete the solution in exponential form. Another reason is that students are not careful and make mistakes

in determining mathematical operations (Asriyani et al., 2020).

6. Exponential function graphs, some students make mistakes when naming the function on a graph. Look at Figure 7 below.

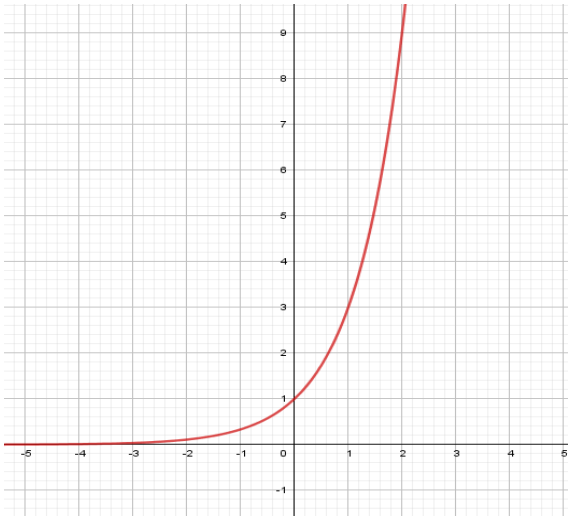

Figure 7. Exponential Function Graph

Subjects S3 and S11 wrote the graph with $f(x) = 3x$, assuming it was a linear function. The correct answer is $f(x) = 3^x$ taking into account the points passed by the curve. If students understand the basic concept of the exponential function $f(x) = ka^x$ and the concept of graphs and Cartesian coordinates, then students can easily find out what function is on the graph. Even S4 answered the question only with the sentence "exponential function". Classified as a coding error, the student's mistake was to do a mathematics assignment, but not check the results again.

Mathematics teachers must pay attention to errors like this so that students do not misunderstand the concept of graphing exponential functions. It was found that the percentage of students' exponential material test results was as follows.

Table 2. Percentage value test

Criteria	Test Score Percentage
Very high	41,7
Tall	8,3
Currently	33,3
Low	16,7
Very low	0

Overall, students made mistakes in their solutions. However, students who make mistakes or carelessness have different levels and locations of errors. The following is a grouping of students based on the level of Newman's Error Analysis.

Table 3. Classification based on Newman error analysis

Subject	Newman's Error Analysis
S1,S2,S7,S10	Reading, Comprehension, dan Transfermation
S3, S4, S8, S11	Transformatio, Process Skills, dan Encoding

Discussion

The study's findings shed light on the many forms of student errors and their causes. Students frequently make errors when operating fractional exponents (Rohaeti et al., 2019) and do not comprehend the notion of exponential functions (Thompson & Carlson, 2017) or their graphs (Kullberg et al., 2017). According to Freeman et al. (2016), students make errors in everything from reading questions to writing responses. Reading errors result from a lack of comprehension of the swimming content or from the inability to recollect previously studied material (Perbowo & Anjarwati, 2017). As a result, it's crucial to briefly review meeting material before moving on to new subject to help students remember it. additionally, misunderstandings the question (comprehension) are where students understand the context of the question but are not understood correctly. This is caused by students' lack of skills to

solve questions so that they are hampered at the stage of understanding the questions (Aksu & Kul, 2016). So students need more practice to develop these skills.

Question transformation errors (transformations) occur in students who already understand the question but fail to operate it. This is caused by a mistake in understanding the concept of operations on numbers (İymen & Duatepe-Paksu, 2015). Students cannot differentiate the operating principles of basic numbers from base numbers. Furthermore, process skill errors at this stage students write ranks in decimal form which can cause the problem solving process to become more complicated. Students do not carry out calculations correctly and change work steps because they do not master them (Skovsmose, 2023). The last error is an error in writing the answer (encoding). This error occurs when students have completed all stages of problem solving but do not include a final conclusion (Salido et al., 2020). This is because students are in a hurry to move on to the next question or the time for completing the questions given has finished.

The inadequate understanding of exponentials by pupils leads to a variety of mistakes. This may occur because students find it challenging to understand exonerating content or because there is exonerating material that has not been adequately grasped. In addition, learning motivation has not been adequately developed, which prevents pupils from wanting to learn more about the subject. Another factor has to do with the 2019 introduction of daring learning. As a result, class X high school students have been doing their homework at home during the COVID-19 pandemic since the eighth grade. One of the students said that he found it challenging to comprehend the mathematical ideas that the teacher was teaching because of his brash study style. This results from unplanned adjustments that must be made to convey the lesson material. Many rights are forced to be ready to face this situation, giving rise to various obstacles.

Conclusion

Based on the findings above, it can be concluded that various forms of student errors were found in operating and drawing exponential function graphs. Many students make carelessness or mistakes when solving class Suggestions from this research are that teachers should really check the results of students' work so that it is clear where the mistakes made by their students are clear. This is useful for improving the form of test questions on student learning outcomes and the teaching and learning process in the future. Regarding the follow-up to this research, teachers should be able to design learning that can help students' abilities and skills in learning mathematics.

Disclosure statement

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